Period 4, Feb 6, 2025

The Law of Cosines

If A, B, and C are the measures of the angles of a triangle, and a, b, and c are the lengths of the sides opposite these angles, then

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C.$$

The square of a side of a triangle equals the sum of the squares of the other two sides minus twice their product times the cosine of their included angle.

Solving an SAS Triangle

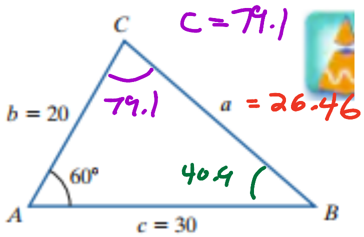
1. Use the Law of Cosines to find the side opposite the given angle.
2. Use the Law of Sines to find the angle opposite the shorter of the two given sides. This angle is always acute.
3. Find the third angle by subtracting the measure of the given angle and the angle found in step 2 from 180°.

Solving an SSS Triangle

1. Use the Law of Cosines to find the angle opposite the longest side.
2. Use the Law of Sines to find either of the two remaining acute angles.
3. Find the third angle by subtracting the measures of the angles found in steps 1 and 2 from 180°.

$60 + 40.9 + C = 180$

$C = 79.1$



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 20^2 + 30^2 - 2(20)(30)\cos 60$

$400 + 900 - 1200(0.5)$

$1300 - 600 =$

$a^2 = 700$

$a = \sqrt{700} = 26.46$

$\frac{20}{\sin B} = \frac{26.46}{\sin 60}$

$\sin B \cdot \frac{20}{\sin B} = 30.55 \cdot \sin B$

$\frac{}{30.55} \quad \frac{}{30.55}$

$0.6547 = \sin B$

$\sin^{-1} 0.6546 = B$

$40.9 = B$

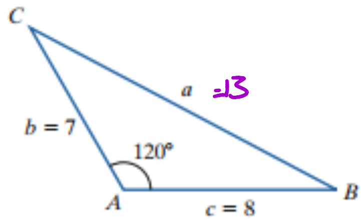


Figure 6.16

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 7^2 + 8^2 - 2\cdot 7\cdot 8\cdot \cos 120$

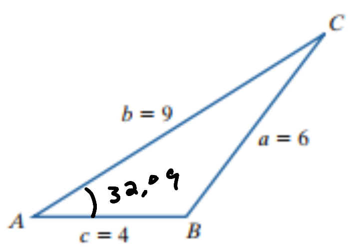
$a^2 = 49 + 64 - 112\cos 120$

$113 - (-56)$

$a^2 = 113 + 56$

$a = \sqrt{169} = 13$

To Find ∠B and ∠C use Law of Sines



Find A

$a^2 = b^2 + c^2 - 2bc\cos A$

$6^2 = 9^2 + 4^2 - 2\cdot 9\cdot 4\cos A$

$36 = 81 + 16 - 72\cos A$

$36 = \cancel{97} - 72\cos A$

$-97 \qquad +\cancel{97}$

$-61 = -\cancel{72}\cos A$

$-72 \qquad -\cancel{72}$

$0.84722 = \cos A$

$\cos^{-1} 0.84722 = A$

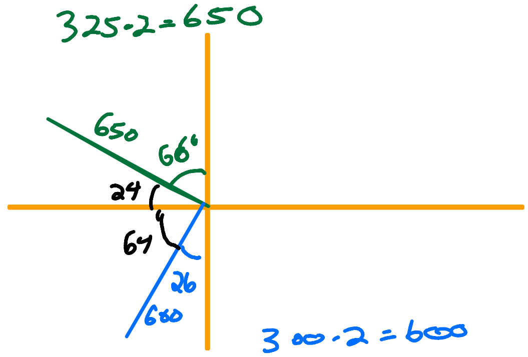
$32.09 = A$

Step 2 Use the Law of Sines to find either of the two remaining acute angles. \

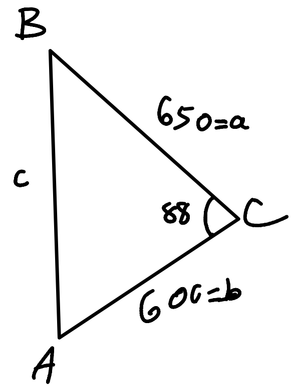
EXAMPLE 3 **An Application of the Law of Cosines**

Two airplanes leave an airport at the same time on different runways. One flies on a bearing of N66°W at 325 miles per hour. The other airplane flies on a bearing of S26°W at 300 miles per hour. How far apart will the airplanes be after two hours?

$325 \cdot 2 = 650$



$300 \cdot 2 = 600$



$$c^2 = a^2 + b^2 - 2ab\cos 88$$
$$c^2 = 650^2 + 600^2 - 2(650)(600)\cos 88$$
$$c^2 = 755278.4$$
$$c = 869.07$$

Heron's Formula for the Area of a Triangle

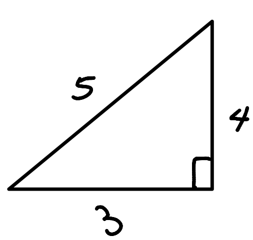
The area of a triangle with sides a, b, and c is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is one-half its perimeter: $s = \frac{1}{2}(a + b + c)$.

Find the area of the triangle with $a = 12$ yards, $b = 16$ yards, and $c = 24$ yards. Round to the nearest square yard.

$$\text{Area} = \tfrac{1}{2} \cdot 3 \cdot 4 = 6$$



Heron's

$$\frac{3+4+5}{2} = \frac{12}{2} = 6$$

$$\text{Area} = \sqrt{6(6-3)(6-4)(6-5)}$$
$$\sqrt{6 \cdot 3 \cdot 2 \cdot 1} = \sqrt{6 \cdot 6} = \sqrt{36} = 6$$

Two sides and an angle (SSA) of a triangle are given. Determine whether the given measurements produce one triangle, two triangles, or no triangle at all. Solve each triangle that results.

$a = 8.2, \ b = 45, \ A = 19^\circ$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Selected the correct choice below and, if necessary, fill in the answer boxes to complete your choice. (Round side lengths to the nearest tenth and angle measurements to the nearest degree as needed.)

- A. There is only one possible solution for the triangle.
  The measurements for the remaining side c and angles B and C are as follows.
  $B \approx$ ☐° $C \approx$ ☐° $c \approx$ ☐
- B. There are two possible solutions for the triangle.
  The measurements for the solution with the the smaller angle B are as follows.
  $B_1 \approx$ ☐° $C_1 \approx$ ☐° $c_1 \approx$ ☐
  The measurements for the solution with the the larger angle B are as follows.
  $B_2 \approx$ ☐° $C_2 \approx$ ☐° $c_2 \approx$ ☐
- C. There are no possible solutions for this triangle.

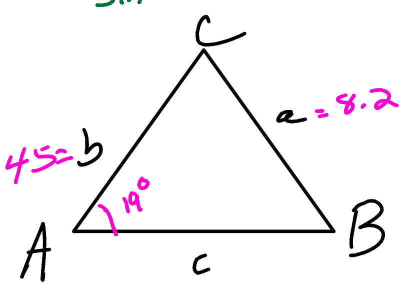
0 △'s

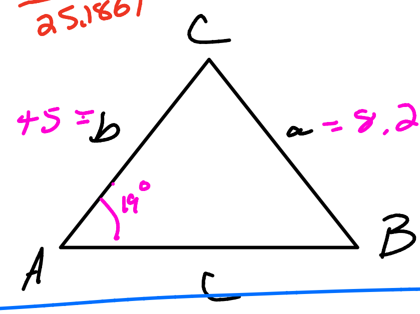
$$\frac{8.2}{\sin 19} = 25.1867$$

$$\sin B \cdot \frac{45}{\cancel{\sin B}} = 25.1867 \cdot \sin B$$

$$\frac{}{25.1867} \qquad \frac{}{25.1867}$$

$$\Rightarrow 1.78 = \sin B$$



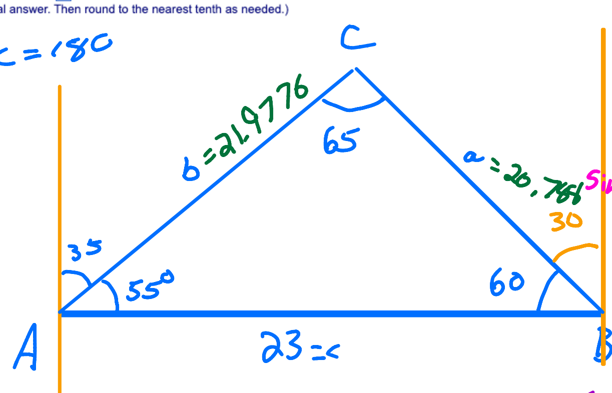


Two fire-lookout stations are 23 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is N35°E and the bearing of the fire from station B is N30°W. How far, to the nearest tenth of a mile, is the fire from each lookout station?

The distance from station B to the fire is ☐ miles.
(Do not round until the final answer. Then round to the nearest tenth as needed.)

$55 + 60 + C = 180$

$C = 65$



$$\frac{23}{\sin 65} = \frac{a}{\sin 55} = \frac{b}{\sin 60}$$

$$\sin 55 \cdot 25.3776 = \frac{a}{\sin 55} \cdot \sin 55$$

$$20.788 = a$$

$$\sin 60 \cdot 25.3776 = \frac{b}{\cancel{\sin 60}} \cdot \sin 60$$

$$21.9776 = b$$

Lighthouse B is 10 miles west of lighthouse A. A boat leaves A and sails 6 miles. At this time, it is sighted from B. If the bearing of the boat from B is N62°E, how far from B is the boat?

The boat is either $\boxed{\sqrt{192.34}}$ miles or $\boxed{13.9}$ miles from lighthouse B, to the nearest tenth of a mile.
(Use descending order. Do not round until the final answer. Then round to the nearest tenth as needed.)

$$\frac{6}{\sin 28} = \frac{10}{\sin C}$$

$$\sin C \cdot 12.78 = \frac{10}{\cancel{\sin C}} \cdot \cancel{\sin C}$$

$$\sin C = \frac{10}{12.78}$$

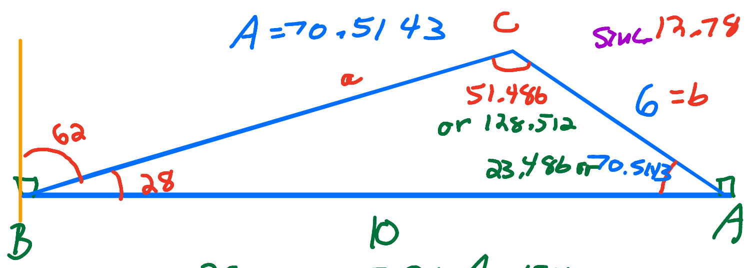
$$\sin C = .78245$$

$$C = \sin^{-1} .78245$$

$$C = 51.486$$

$$28 + 51.486 + A = 180$$

$$A = 70.5143$$



$$28 + 128.512 + A = 180$$

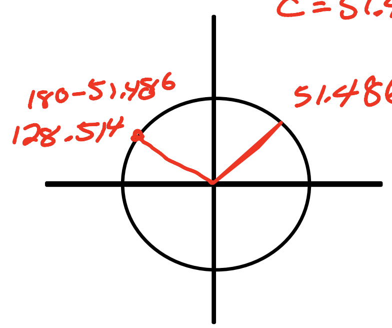
$$A = 23.486$$

$$\frac{6}{\sin 28} = \frac{a}{\sin 70.5143}$$

$$12.78 = \frac{a}{\sin 70.5143}$$

$$(12.78)(\sin 70.5143) = a$$

$$12.048 = a$$



$$\frac{6}{\sin 28} = \frac{a}{\sin 23.486}$$

$$\sin 23.486 \cdot 12.78 = \frac{a}{\cancel{\sin 23.486}} \cdot \cancel{\sin 23.486}$$

$$5.09 = a$$

Two buildings of equal height are 840 feet apart. An observer on the street between the buildings measures the angles of elevation to the tops of the buildings as 30° and 42°. How high, to the nearest foot, are the buildings?

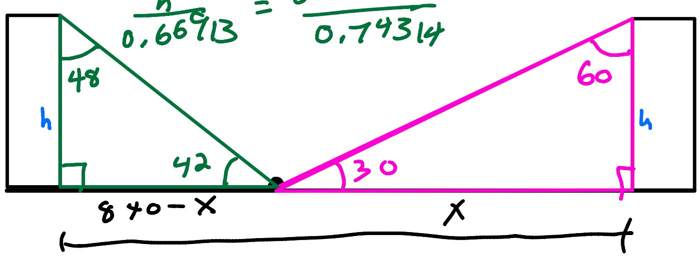
The buildings are about 295 feet high.
(Round to the nearest foot as needed.)

$$\frac{h}{\sin 30} = \frac{x}{\sin 60}$$

$$\frac{h}{0.5} = \frac{x}{0.866}$$

$$\frac{h}{\sin 42} = \frac{840-x}{\sin 48}$$

$$\frac{h}{0.66913} = \frac{840-x}{0.74314}$$



$$\frac{\cancel{0.866}h}{\cancel{0.866}} = \frac{0.5x}{0.866}$$

$$\frac{\cancel{0.74314}h}{\cancel{0.74314}} = \frac{0.66913(840-x)}{0.74314}$$

$$h = 0.57735(511.816)$$

$$h = 295.5$$

$$h = .57735x$$

$$h = 0.9004(840-x)$$

$$0.9004(840-x) = 0.57735x$$

$$756.336 - 0.9004x = 0.57735x$$

$$+0.9004x \quad 0.9004x$$

$$\frac{756.336}{1.47775} = \frac{\cancel{1.47775}x}{\cancel{1.47775}}$$

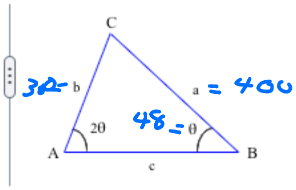
$$x = 511.816$$

Use the given measurements to solve the triangle. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

a = 400, b = 300

$$48+96+c=180$$

$$c=36$$



The measure of angle B is approximately 48 °.
(Round to the nearest degree.)

The measure of angle A is approximately 96 °. $= 2 \cdot 48$
(Round to the nearest degree.)

The measure of angle C is approximately 36 °.
(Round to the nearest degree.)

The length of c is approximately 236.4 .
(Round to the nearest tenth.)

$$\frac{400}{\sin 96} = \frac{c}{\sin 36}$$

Find C

$$\frac{400}{\sin 2\theta} = \frac{300}{\sin\theta}$$

$$300\sin 2\theta = 400\sin\theta$$

$$300(2\sin\theta\cos\theta) = 400\sin\theta$$

$$600\sin\theta\cos\theta - 400\sin\theta = 0$$

$$200\sin\theta(3\cos\theta - 2) = 0$$

$200\sin\theta = 0$ or $3\cos\theta - 2 = 0$

$\sin\theta = 0$ $\qquad$ $\cos\theta = \frac{2}{3}$

$\theta = 0$ $\qquad$ $\theta = \cos^{-1}\frac{2}{3}$

$$\theta = 48.1897$$

Two sides and an angle (SSA) of a triangle are given. Determine whether the given measurements produce one triangle, two triangles, or no triangle at all. Solve each triangle that results.

$a = 16$, $b = 17$, $A = 64°$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice. (Round side lengths to the nearest tenth and angle measurements to the nearest degree as needed.)

- A. There is only one possible solution for the triangle.
  The measurements for the remaining side c and angles B and C are as follows.
  $B \approx$ ☐° $C \approx$ ☐° $c \approx$ ☐
- B. There are two possible solutions for the triangle.
  The measurements for the solution with the the smaller angle B are as follows.
  $B_1 \approx$ 73° $C_1 \approx$ 43° $c_1 \approx$ 12.1
  The measurements for the solution with the the larger angle B are as follows.
  $B_2 \approx$ 107° $C_2 \approx$ 9° $c_2 \approx$ 2.8
- C. There are no possible solutions for this triangle.

$$\frac{16}{\sin 64} = \frac{17}{\sin B}$$

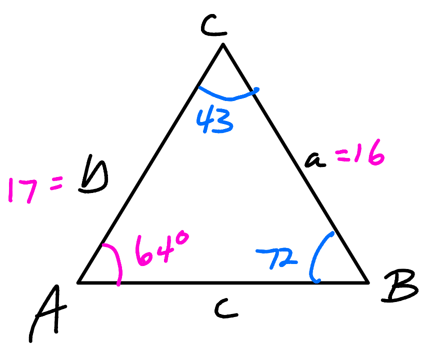
$$17.802 = \frac{17}{\sin B}$$

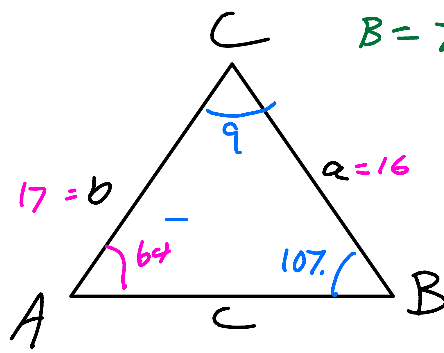
$$\sin B = \frac{17}{17.802} = .95497 =$$

$$B = 72.7°$$

$$B = 180 - 72.7$$

$$B = 107.3$$





$$\frac{c}{\sin 43} = \frac{16}{\sin 64}$$

$$\frac{c}{\sin 43} = 17.802$$

$$c = 17.802 \sin 43$$

$$c = 12.1409$$

$$\frac{c}{\sin 9} = \frac{16}{\sin 64}$$

$$c = 2.8$$

Find the area of the triangle with $a = 12$ yards, $b = 16$ yards, and $c = 24$ yards. Round to the nearest square yard.

Heron's

$$\frac{12+16+24}{2} = \frac{52}{2} = 26$$

$$\text{Area} = \sqrt{26(26-12)(26-16)(26-24)}$$

$$\sqrt{26 \cdot 14 \cdot 10 \cdot 2} = \sqrt{7280}$$

$$A = \sqrt{7280}$$

$$\text{Area} = 85.323$$